AC Length Constant

Along a neurite $\frac{V_x}{V_0} = e^{-x/\lambda_\omega}$ where $\lambda_\omega = \sqrt{Z_m / r_a}$, Z_m = membrane impedance of a unit length of neurite, and r_a = resistance of a unit length of cytoplasm.

Assume f is sufficiently high that transmembrane current can be regarded as entirely capacitive ($f > 5/2\pi\tau_m$, i.e. for a neuron with τ_m = 30 ms, f > 25 Hz). Then $Z_m \approx 1/j\omega C$ where C = capacitance of a unit length of neurite.

Thus at high frequencies $\lambda_\omega \approx 1/\sqrt{j\omega C r_a}$ and $\frac{V_x}{V_0} \approx e^{-x\sqrt{j\omega C r_a}} = e^{-x(1+j)\sqrt{\frac{\omega C r_a}{2}}}$. The real part of the exponent is the signal attenuation, and the imaginary part is the phase shift, so $\left|\frac{V_x}{V_0}\right| \approx e^{-x\sqrt{\frac{\omega C r_a}{2}}}$.

Substituting $r_a = R_i/\pi a^2$ and $C = 2\pi a C_m$, where a = radius of neurite, R_i = cytoplasmic resistivity in Ω cm, and C_m = specific membrane capacitance in μf/cm^2, we have $\frac{r_a C}{2} = \frac{1}{2} \cdot \frac{R_i}{\pi a^2} \cdot 2\pi a C_m = \frac{R_i C_m}{a}$. Therefore $\left|\frac{V_x}{V_0}\right| \approx e^{-x\sqrt{\frac{\omega R_i C_m}{a}}}$ and the AC space constant is $\lambda_\omega \approx \sqrt{\frac{2}{r_a C\omega}} = \sqrt{\frac{a}{R_i C_m \omega}} = \sqrt{\frac{a}{2\pi f R_i C_m}}$. If a is in μm, f in Hz, R_i in Ω cm, and C_m in μf/cm^2, the numerical result must be multiplied by 10^5 to convert it to μm.

Example: consider a neurite with radius of 1 μm, R_m = 50,000 Ω cm^2, R_i = 100 Ω cm, and C_m = 1 μf/cm^2. The membrane time constant is 50 ms, so the frequency at which membrane resistive and capacitive current are equal is ~ 3.2 Hz. The DC length constant is $\lambda_{DC} = \sqrt{r_m/r_a}$, which turns out to be ~ 1500 microns. The AC length constant at 100 Hz is only 400 microns, roughly 4 times shorter.

Addendum: NEURON uses R_a to signify R_i, and neurite diameter is specified rather than radius. Thus in the context of NEURON models it is more convenient to rewrite the AC length constant formula as $\lambda_f \approx \frac{1}{2}\sqrt{\frac{d}{\pi f R_a C_m}}$.